Rewetting Analysis of a PWR Slab Fuel Using an Improved Lumped Model

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Outline

1 – Introduction
2 – Literature Review
3 – Proposed Method
4 – Temperature Profiles
5 – Results and Discussion
6 – Conclusions and Future Work
Introduction

- **Severe Accidents:**
  - Three Mile Island (TMI) – USA, 1979
  - Chernobyl – Ukraine, 1986
  - Fukuishima – Japan, 2011

Illustration of the TMI-2 core after partial meltdown.
Loss of Coolant Accident (LOCA) – Design Basis Accident (DBA)

• **Definition:**
  • postulated design basis accident in order to assure reactor’s core integrity given its occurrence.

• **Premisses:**
  • Shut down reactor in the event occurrence;
  • Total failure in the flow of a loop’s cold leg.

• **Phases:**
  • Blowdown phase;
  • Bypass phase;
  • Refill phase;
  • Reflooding phase.
The Physical Problem:
• Nuclear fuel plate rewetting;
• Bidimensional problem;
• Axissymmetric longitudinal rewetting;
• Two discontinuous rewetting regions.

Objectives:
• Average temperature profiles;
• Peclet numbers;
• $Q_{\text{crit}}$ values;
• Biot number and initial wall temperature behavior and influence on the rewetting.
Literature Review
Rewetting Phenomena

- **Yamanouchi (1968):**
  - Experimental.
- **Duffery and Porthouse (1973):**
  - Experimental and analytical 1D e 2D.
- **Sun et al. (1973):**
  - Analytical 1D; 3 regions.
- **Yeh (1975):**
  - Exact solution;
- **Yeh (1980):**
  - Exact solution;
- **Thomas (1988):**
  - Wiener-Hopf technique.
- **Olek (1989):**
  - Wiener-Hopf technique.
- **Ferugli et al. (1991):**
  - Coupled solution.
- **Sahu et al. (2008):**
  - Heat Balance Integral Method;
Literature Review
Lumped Model

• Mennig and Ozisik (1985):
  • Trapezoidal approximations.
• Cotta et al. (1990):
  • Trapezoidal approximations.
• Cotta and Mikhailov (1997):
  • Hermite integrals.
• Corrêa and Cotta (1998):
  • Diffusion.
• Régis et al. (2000):
  • Transient fuel rod.
• Su and Cotta (2001):
  • LWR core dynamics.
Literature Review
Lumped Model

• Hermite approximations:

\[
\int_{a}^{b} y(x) \, dx = \sum_{\nu=0}^{a} C_{\nu}(a, b) h^{\nu+1} y^{[\nu]}(a) + \sum_{\nu=0}^{b} C_{\nu}(b, a) (-1)^{\nu} h^{\nu+1} y^{[\nu]}(b) + O(h^{a+b+3}),
\]

\[
C_{\nu}(a, b) = \frac{(a+1)!(a+b+1-\nu)!}{(\nu+1)!(a-\nu)!(a+b+2)!}
\]

\[h = b - a\]
Literature Review

Lumped Model

- Hermite approximations:

\[
H_{0,0} \rightarrow \int_{a}^{b} y(x)dx \approx \frac{h}{2}(y(a) + y(b))
\]

\[
H_{1,0} \rightarrow \int_{a}^{b} y(x)dx \approx \frac{2h}{3}(y(a) + y(b)) + \frac{h^2}{6}y'(b)
\]

\[
H_{1,1} \rightarrow \int_{a}^{b} y(x)dx \approx \frac{h}{2}(y(a) + y(b)) + \frac{h^2}{12}(y'(b) - y'(a))
\]
Proposed Method

Physical modelling

Schematics of a top-spray rewetting.
Source: SAHU, S. K., DAS, P. K., BHATTACHARYYA, S.

Physical modelling

Lateral view of a top-spray rewetting.
Assumptions:

1. Axisymmetric and quasi-stead flow;
2. Constant internal heat generation;
3. Constant coolant temperature and equal to the saturation temperature;
4. Constant steam temperature;
5. Initial steam temperature equal to the wall temperature.
Proposed Method
Differential Equations

• Differential Equation for the Dimensionless Temperature:

\[
\frac{\partial^2 \theta_f(\bar{r}, \bar{z})}{\partial \bar{r}^2} + \frac{\partial^2 \theta_f(\bar{r}, \bar{z})}{\partial \bar{z}^2} + Pe \frac{\partial \theta_f(\bar{r}, \bar{z})}{\partial \bar{z}} + Q = 0;
\]

• Boundary Conditions:

\[
\left. \frac{\partial \theta_f(\bar{r}, \bar{z})}{\partial \bar{r}} \right|_{\bar{r}=0} = 0, \text{ for } -\infty < \bar{z} < \infty;
\]
\[
- \left. \frac{\partial \theta_f(\bar{r}, \bar{z})}{\partial \bar{r}} \right|_{\bar{r}=1} = Bi_1 \theta_f(\bar{r}, \bar{z}), \text{ for } -\infty < \bar{z} \leq 0;
\]
\[
\left. \frac{\partial \theta_f(\bar{r}, \bar{z})}{\partial \bar{r}} \right|_{\bar{r}=R_{fo}} = Bi_2 \left( \theta_f(\bar{r}, \bar{z}) - (1 + \theta_1) \right), \text{ for } 0 \leq \bar{z} < \infty.
\]
Proposed Method

Differential Equations

• Differential Equations for the Dimensionless Average Temperature:

\[
\frac{d^2 \theta_{fav}(\bar{z})}{d\bar{z}^2} + Pe \frac{d\theta_{fav}(\bar{z})}{d\bar{z}} + Q - Bi_1 \theta_f(1, \bar{z}) = 0, \text{ for } -\infty < \bar{z} \leq 0;
\]

\[
\frac{d^2 \theta_{fav}(\bar{z})}{d\bar{z}^2} + Pe \frac{d\theta_{fav}(\bar{z})}{d\bar{z}} + Q - Bi_2 \left( \theta_f(1, \bar{z}) - (1 + \theta_1) \right) = 0, \text{ for } 0 \leq \bar{z} < +\infty;
\]
Temperature Profiles
Classical Lumped Parameters Solution

- **Functions Approximation:**

  \[\theta_f(0, \bar{z}) = \theta_f(1, \bar{z}) = \theta_{fav}(\bar{z})\]

- **Dimensionless Average Temperature Profile for the CLSA:**

  \[
  \theta_{fav}(\bar{z}) = \frac{Q}{Bi_1} + \left(1 - \frac{Q}{Bi_1}\right) e^{\frac{1}{2}(Pe - \sqrt{4Bi_1Pe^2})\bar{z}}, \text{ for } -\infty < \bar{z} \leq 0;
  \]

  \[
  \theta_{fav}(\bar{z}) = 1 + \theta_1 + \frac{Q}{Bi_2} + \left(\theta_1 + \frac{Q}{Bi_2}\right) e^{\frac{1}{2}(Pe - \sqrt{4Bi_1Pe^2})\bar{z}}, \text{ for } 0 \leq \bar{z} < +\infty.
  \]
Temperature Profiles

Improved Lumped Parameters Solution $H_{0,0}/H_{0,0}$

- Functions Approximation:

\[ H_{0,0} \rightarrow \int_0^1 \theta_f(\bar{r}, \bar{z})d\bar{z} \approx \frac{1}{2}(\theta_f(0, \bar{z}) + \theta_f(1, \bar{z})) \quad H_{0,0} \rightarrow \int_0^1 \frac{\partial \theta_f(\bar{r}, \bar{z})}{\partial \bar{r}}d\bar{z} \approx \frac{1}{2} \left( \frac{\partial \theta_f(0, \bar{z})}{\partial \bar{r}} + \frac{\partial \theta_f(1, \bar{z})}{\partial \bar{r}} \right) \]

- Dimensionless Average Temperature Profile for the ILSA $H_{0,0}/H_{0,0}$:

\[
\theta_{f_{av}}(\bar{z}) = \frac{(4 + Bi_1)Q}{4Bi_1} + \left(1 - \frac{(4 + Bi_1)Q}{4Bi_1}\right) e^{\frac{1}{2}\left(\frac{Pe}{\sqrt{4 + Bi_1 + Pe^2}}\right)\bar{z}}, \text{ for } -\infty < \bar{z} \leq 0; \\
\theta_{f_{av}}(\bar{z}) = 1 + \theta_1 + \frac{Q}{Bi_2} + \frac{Q}{4} + \left(\theta_1 + \frac{Q}{Bi_2} + \frac{Q}{4}\right) e^{\frac{1}{2}\left(-\frac{Pe}{\sqrt{4 + Bi_1 + Pe^2}}\right)\bar{z}}, \text{ for } 0 \leq \bar{z} < +\infty.
\]
Temperature Profiles

Improved Lumped Parameters Solution \( H_{1,1}/H_{0,0} \)

• Functions Approximation:

\[
H_{1,1} \rightarrow \int_0^1 \theta_f(\bar{r}, \bar{z})d\bar{z} \approx \frac{1}{2} (\theta_f(0, \bar{z}) + \theta_f(1, \bar{z})) + \quad H_{0,0} \rightarrow \int_0^1 \frac{\partial \theta_f(\bar{r}, \bar{z})}{\partial \bar{r}} d\bar{z} \approx \frac{1}{2} \left( \frac{\partial \theta_f(0, \bar{z})}{\partial \bar{r}} + \frac{\partial \theta_f(1, \bar{z})}{\partial \bar{r}} \right) + \frac{1}{12} \left( \frac{\partial \theta_f(1, \bar{z})}{\partial \bar{r}} - \frac{\partial \theta_f(0, \bar{z})}{\partial \bar{r}} \right)
\]

• Dimensionless Average Temperature Profile for the ILSA \( H_{1,1}/H_{0,0} \):

\[
\theta_{f_{av}}(\bar{Z}) = \frac{(3+Bi_1)Q}{3Bi_1} + \left(1 - \frac{(3+Bi_1)Q}{3Bi_1}\right) e^{\frac{1}{2} \left(-Pe + \sqrt{\frac{12Bi_1}{3+Bi_1} + Pe^2}\right) \bar{Z}}, \text{ for } -\infty < \bar{Z} \leq 0;
\]

\[
\theta_{f_{av}}(\bar{Z}) = 1 + \theta_1 + \frac{Q}{Bi_2} + \frac{Q}{3} + \left(\theta_1 + \frac{Q}{Bi_2} + \frac{Q}{3}\right) e^{\frac{1}{2} \left(-Pe - \sqrt{\frac{12Bi_1}{3+Bi_1} + Pe^2}\right) \bar{Z}}, \text{ for } 0 \leq \bar{Z} < +\infty.
\]
Results and Discussion

Physical Properties

\[ k_f = 2.163\ W/m.K \]
\[ \rho_f = 10,970\ kg/m^3 \]
\[ R_{fo} = 4.1\ mm \]
\[ q^{\infty} = 318.121\ MW/m^3 \]

Dimensionless Variables

<table>
<thead>
<tr>
<th>Dimensionless Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bi_1</td>
<td>10.0, 1.0 and 0.1</td>
</tr>
<tr>
<td>Bi_2</td>
<td>0.001Bi_1</td>
</tr>
<tr>
<td>Q</td>
<td>1, 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4} and 10^{-5}</td>
</tr>
<tr>
<td>\theta_1</td>
<td>0.5, 0.9 and 1.5</td>
</tr>
</tbody>
</table>

It is assumed that after 100 seconds after a LOCA, the reflooding phase starts and the remaining power inside the core is around 3.5%
Results and Discussion

Fuel plate dimensionless average temperature profile variation $H_{1,1}$ approximation to the fuel temperature and $H_{0,0}$ to the derivative)
Results and Discussion

Fuel plate dimensionless average temperature profile comparison of the different formulations with the heat source parameter of $10^{-5}$.
Results and Discussion

Fuel plate dimensionless average temperature profile comparison of the different formulations with the heat source parameter of $10^{-1}$. 
## Results and Discussion

**PECLET NUMBERS WITH $Bi_1 = 10.0$.**

<table>
<thead>
<tr>
<th>Formulation</th>
<th>Heat Source Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$10^{-5}$</td>
</tr>
<tr>
<td>CLSA</td>
<td>3.65</td>
</tr>
<tr>
<td>ILSA</td>
<td>$H_{0,0}/H_{0,0}$</td>
</tr>
<tr>
<td>ILSA</td>
<td>$H_{1,1}/H_{0,0}$</td>
</tr>
<tr>
<td>Sahu et al. [5]</td>
<td></td>
</tr>
</tbody>
</table>
Results and Discussion

Peclet number variation for diverse heat source parameters and Biot numbers for the ILSA $H_{1,1}/H_{0,0}$ formulation.
Peclet number variation for diverse heat source parameters and formulations for $Bi_1 = 10.0$ and $\theta_1 = 0.5$. 
Results and Discussion

Peclet number variation for diverse heat source parameters and formulations for $Bi_1 = 0.1$ and $\theta_1 = 0.5$. 
### Results and Discussion

**$Q_{crit}$ VALUES FOR THE CLASSICAL AND IMPROVED LUMPED FORMULATIONS**

<table>
<thead>
<tr>
<th>Formulation</th>
<th>Biot Number</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10.0</td>
</tr>
<tr>
<td>Classical lumped</td>
<td>0.295</td>
</tr>
<tr>
<td>Improved lumped \ $H_{0,0}/H_{0,0}$</td>
<td>0.1514</td>
</tr>
<tr>
<td>Improved lumped \ $H_{1,1}/H_{0,0}$</td>
<td>0.1349</td>
</tr>
<tr>
<td>Sahu et al. [5]</td>
<td>0.1445</td>
</tr>
</tbody>
</table>
Conclusions and Future Work

Conclusions:
• Convergence between CLSA and ILSA formulations for low Biot Numbers;
• Presumably better results than the Heat Balance Integral Method;
• Less difference between formulations for smaller $Q$ values;
• Less computational time;
• Method adaptable to other physical configurations.

Future Work:
• Fuel rod rewetting analysis;
• Annular fuel rewetting analysis
• Inclusion of a third region in the interface between wet and dry regions.
Thank you!